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Coherently manipulating two-qubit quantum information using a pair of simultaneous laser pulses

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Abstract. – Several sequential operations are usually needed for implementing controlled quantum gates and generating entanglement between a pair of quantum bits. Based on the conditional quantum dynamics for a two-ion system beyond the Lamb-Dicke limit, here we propose a theoretical scheme for manipulating two-qubit quantum information, *i.e.*, implementing a universal two-qubit quantum gate and generating a two-qubit entangled state, by using a pair of simultaneous laser pulses. Neither the Lamb-Dicke approximation nor the auxiliary atomic level are required. The experimental realizability of this simple approach is also discussed.

As first suggested in [1], one of the most promising scenarios for implementing a practical quantum information processor is a system of laser-cooled trapped ions, due to its long coherence time [2]. In this system the qubit (*i.e.*, the elementary unit of quantum information) is encoded by two internal levels of each trapped cold ion and can be manipulated individually by using laser pulses. As explained below, a third auxiliary internal level of the ion is also required. Several key features of the proposal in [1], including the production of entangled states and the implementation of quantum controlled operations between a pair of trapped ions, have already been experimentally demonstrated (*e.g.*, [3–6]). Meanwhile, several alternative theoretical schemes (*e.g.*, [7–12]) have also been developed for overcoming various difficulties in realizing a practical ion-trap quantum information processor.

Auxiliary transitions between the encoded atomic levels and the auxiliary ones [1,3,5] and a series of laser pulses [1,9–11] are usually required in various previous schemes for manipulating two-qubit quantum information. For example, a *three*-step operation is required in [13], *five* laser pulses in [1, 9], *six* pulses in ref. [10], and *seven* pulses are needed in ref. [11] in order to perform a quantum controlled-NOT (CNOT) logic gate. The first aim of the present work is to propose an efficient scheme for realizing quantum logic operations between a pair of trapped cold ions by *a single-step operation.* The Lamb-Dicke (LD) approximation, which requires that the coupling between the external and internal degrees of freedom of the ion be very weak, is made in *almost all* of the previous schemes (*e.g.*, [4, 6, 8, 11, 12]) in order to simplify the laser-ion interaction Hamiltonian. However, the quantum motion of the trapped ions is *not* limited to the LD regime [14]. Therefore, it is important to manipulate quantum information stored in trapped ions outside the LD limit.

The entanglement between different particles is a growing focus of activity in quantum physics [15], because of experiments on non-local features of quantum mechanics and the development of quantum information physics. Especially, entanglement plays a central role in quantum parallelism. The quantum entanglement of two and four trapped cold ions have been generated experimentally [5, 6] in the LD limit. The second aim of the present work is to show how to deterministically produce the entangled states of two trapped ions outside the LD regime. The third aim is to achieve this without auxiliary internal levels.

In summary, here we propose a scheme for manipulating quantum information (*i.e.*, realizing quantum controlled operations and generating entanglement) of two trapped ions: i) *beyond* the LD limit, ii) *without* needing any auxiliary atomic level, and iii) by using a *single-step operation.*

We consider an array of N two-level cold ions of mass M confined to move in the z-direction of a Paul trap of frequency ν. The ions are cooled down to very low temperatures and may perform small oscillations around their equilibrium position z_{i0} ($i = 1, 2, \ldots, N$), due to their mutual repulsive Coulomb force. Each ion is assumed to be individually addressed by a separate laser beam. Similarly to ref. [16], we consider the case where an arbitrary pair (labeled by $j = 1, 2$) of the N trapped cold ions are illuminated independently by two weak travelling laser fields with frequencies ω_j. The Hamiltonian corresponding to this situation is

$$\begin{aligned}\hat{H}(t) &= \hbar\omega_0 \sum_{j=1}^{2} \frac{\hat{\sigma}_j^z}{2} + \sum_{l=0}^{N-1} \hbar\nu_l \left(\hat{b}_l^\dagger \hat{b}_l + \frac{1}{2}\right) + \\ &+ \frac{\hbar}{2} \sum_{j=1}^{2} \left\{ \Omega_j \hat{\sigma}_j^+ \exp\left[\sum_{l=0}^{N-1} i\eta_j^l (\hat{b}_l^\dagger + \hat{b}_l) - i\omega_j t - i\phi_j\right] + \text{H.c.} \right\}. \qquad (1)\end{aligned}$$

Here, ν_l ($l = 0, 1, \ldots, N-1$) is the frequency of the l-th mode collective vibrational motion, and the LD parameter η_j^l accounts for the coupling strength between the internal state of the j-th ion and the l-th mode of the collective vibration. $\hat{b}_l^\dagger$ and $\hat{b}_l$ are the relevant ladder operators. Ω_j is the carrier Rabi frequency, which describes the coupling strength between the laser and the j-th ion and is proportional to the strength of the applied laser. $\hat{\sigma}_j^z$ and $\hat{\sigma}_j^\pm$ are Pauli operators, $\hbar\omega_0$ is the energy separation of two internal states $|g\rangle$ and $|e\rangle$ of the ion, and ϕ_j is the initial phase of the applied laser beam. Expanding the above Hamiltonian in terms of creation and annihilation operators of the normal modes, we have

$$\begin{aligned}\hat{H} &= \frac{\hbar}{2} \sum_{j=1,2} \left\{ \Omega_j \hat{\sigma}_j^+ \prod_{l=0}^{N-1} \times \right. \\ &\left. \times \left[e^{-(\eta_j^l)^2/2} \sum_{m,n=0}^{\infty} \frac{\left(i\eta_j^l\right)^{m+n}}{m!n!} \hat{b}_l^{\dagger m} \hat{b}_l^n \exp\left[i(m-n)\nu_l t + i(\delta_j t - \phi_j)\right] \right] + \text{H.c.} \right\} \qquad (2)\end{aligned}$$

in the interaction picture. Here, $\delta_j = \omega_j - \omega_0$ is the detuning between the laser and the ion. Without any loss of generality, we set the frequencies of the applied laser beams as $\omega_j = \omega_0 - k_j\nu$ with $\nu = \nu_0$ being the frequency of the center-of-mass (CM) vibrational mode, $k_2 = 0$, and $k_1 = 1, 2, \ldots$. Like the procedure described in [16, 17], we then make the usual rotating wave approximation (RWA). For small values of k_1, the excitations of the higher l-th ($l \geq 1$) vibrational modes can be safely neglected and the following effective Hamiltonian:

$$\hat{H} = \frac{\hbar}{2} \sum_{j=1,2} \left\{ \Omega_j \hat{F}_j \hat{\sigma}_j^+ \exp\left[-\frac{\eta_j^2}{2} - i\phi_j\right] \sum_{n=0}^{\infty} \frac{\left(i\eta_j\right)^{2n+k_j} \hat{a}^{\dagger n} \hat{a}^{n+k_j}}{n!(n+k_j)!} + \text{H.c.} \right\} \qquad (3)$$

can be obtained. Here, $\hat{a} = \hat{b}_0$, $\hat{a}^\dagger = \hat{b}_0^\dagger$ and $\eta_j = \eta_j^0$ are the boson operators and the LD parameter related to the CM mode, respectively. The operator function $\hat{F}_j = \prod_{l=1}^{N-1} \exp[-(\eta_j^l)^2/2] \times \sum_{n=0}^{\infty} (i\eta_j^l)^{2n} \hat{b}_l^{\dagger n} \hat{b}_l^n/(n!)^2$ is irrelevant [16, 17, 19] in the weak-excitation regime ($\Omega_j \ll \nu$). Therefore, we may let $\hat{F}_j = \hat{I}$ and only label the CM mode excitations.

It is very important to stress that, to the lowest order of the LD parameter η_j, the effective Hamiltonian (3) reduces to that in previous works [4, 6, 8, 12, 17] under the usual LD approximation: $(m + 1/2)\eta_j^2 \ll 1$. Here, m is the occupation number of the Fock state of the CM vibrational quanta. We now analytically solve the quantum dynamical problem associated with the Hamiltonian (3) *without* performing the LD approximation. For simplicity, the information bus (*i.e.*, the CM vibrational mode of the ions) is assumed to be prepared beforehand in an arbitrary pure quantum state, *e.g.*, the Fock state $|m\rangle$. The Hamiltonian (3) implies that the laser pulse applied to the second ion does not excite the CM vibrational state but only rotates the atomic state of the second ion. If the condition $k_1 > m$ is satisfied, then the ground state $|g_1\rangle$ of the first ion will not evolve. Therefore, $\{|m\rangle|g_1\rangle|g_2\rangle, |m\rangle|g_1\rangle|e_2\rangle\}$ and $\{|m\rangle|e_1\rangle|g_2\rangle, |m\rangle|e_1\rangle|e_2\rangle, |m+k_1\rangle|g_1\rangle|g_2\rangle, |m+k_1\rangle|g_1\rangle|e_2\rangle\}$ form two different invariant subspaces of the dynamics ruled by the Hamiltonian (3) with $k_2 = 0$, $k_1 > m$. After a direct derivation, we obtain the exact time evolutions in these subspaces:

$$\begin{cases} |m\rangle|g_1\rangle|g_2\rangle \longrightarrow \cos(\tilde{\alpha}_2 t)|m\rangle|g_1\rangle|g_2\rangle - ie^{-i\phi_2}\sin(\tilde{\alpha}_2 t)|m\rangle|g_1\rangle|e_2\rangle, \\ |m\rangle|g_1\rangle|e_2\rangle \longrightarrow \cos(\tilde{\alpha}_2 t)|m\rangle|g_1\rangle|e_2\rangle - ie^{i\phi_2}\sin(\tilde{\alpha}_2 t)|m\rangle|g_1\rangle|g_2\rangle, \\ |m\rangle|e_1\rangle|g_2\rangle \longrightarrow E_1(t)|m+k_1\rangle|g_1\rangle|g_2\rangle + E_2(t)|m+k_1\rangle|g_1\rangle|e_2\rangle + \\ \qquad\qquad + E_3(t)|m\rangle|e_1\rangle|g_2\rangle + E_4(t)|m\rangle|e_1\rangle|e_2\rangle, \\ |m\rangle|e_1\rangle|e_2\rangle \longrightarrow F_1(t)|m+k_1\rangle|g_1\rangle|g_2\rangle + F_2(t)|m+k_1\rangle|g_1\rangle|e_2\rangle + \\ \qquad\qquad + F_3(t)|m\rangle|e_1\rangle|g_2\rangle + F_4(t)|m\rangle|e_1\rangle|e_2\rangle, \end{cases} \tag{4}$$

with

$$\begin{aligned} E_1(t) &= (-i)^{k_1+1}\frac{e^{i\phi_1}}{\Delta}[\sin(\lambda_+ t) - \sin(\lambda_- t)], \\ E_4(t) &= -ie^{-i\phi_2}\frac{\rho^2}{\Delta}\left[\frac{\sin(\lambda_+ t)}{\zeta_+} - \frac{\sin(\lambda_- t)}{\zeta_-}\right], \\ E_2(t) &= (-i)^{k_1} e^{i(\phi_1-\phi_2)}\left(\frac{\alpha_1\rho^2 + \tilde{\gamma}_2\zeta_+\rho}{\lambda_+\zeta_+\Delta}\right)[\cos(\lambda_+ t) - \cos(\lambda_- t)], \\ E_3(t) &= \frac{\rho^2}{\Delta}\left[\frac{\cos(\lambda_+ t)}{\zeta_+} - \frac{\cos(\lambda_- t)}{\zeta_-}\right]; \\ F_1(t) &= (-i)^{k_1} e^{i(\phi_1+\phi_2)}\frac{\rho}{\Delta}[\cos(\lambda_+ t) - \cos(\lambda_- t)], \\ F_4(t) &= \frac{\rho^2}{\Delta}\left[\frac{\cos(\lambda_+ t)}{\zeta_+} - \frac{\cos(\lambda_- t)}{\zeta_-}\right], \\ F_2(t) &= (-i)^{k_1+1} e^{i\phi_1}\left(\frac{\alpha_1\rho^2 + \tilde{\gamma}_2\zeta_+\rho}{\lambda_+\zeta_+\Delta}\right)[\sin(\lambda_+ t) - \sin(\lambda_- t)], \\ F_3(t) &= -ie^{i\phi_2}\frac{\rho^2}{\Delta}\left[\frac{\sin(\lambda_+ t)}{\zeta_+} - \frac{\sin(\lambda_- t)}{\zeta_-}\right]. \end{aligned}$$

Here,

$$\begin{aligned}
\rho &= \alpha_1(\tilde{\alpha}_2+\tilde{\gamma}_2), \qquad \lambda_\pm = \sqrt{\frac{\Lambda\pm\Delta}{2}}, \qquad \Lambda = \tilde{\alpha}_2^2+\tilde{\gamma}_2^2+2\alpha_1^2,\\
\Delta^2 &= \Lambda^2-4\big(\tilde{\alpha}_2\tilde{\gamma}_2-\alpha_1^2\big)^2, \qquad \zeta_\pm = \lambda_\pm^2-\tilde{\alpha}_2^2-\alpha_1^2;\\
\alpha_j &= \Omega_m^{(k_j)}, \qquad \Omega_m^{(k_j)} = \frac{\Omega_j(\eta_j)^{k_j}\exp\big[-\eta_j^2/2\big]}{2}\sqrt{\frac{(m+k_j)!}{m!}}\sum_{n=0}^{m}\frac{\big(-i\eta_j\big)^{2n}}{(n+k_j)!}\binom{n}{m},\\
\tilde{\alpha}_j &= \Omega_m^{(0)}, \qquad \tilde{\gamma}_j = \Omega_{m+k_l}^{(0)}, \qquad j,l=1,2,\ j\neq l.
\end{aligned}$$

Note that the above evolutions are performed by a single-step operation by applying, separately to two ions, a pair of simultaneous laser pulses with different frequencies. The exact dynamical evolution for other cases can also be derived exactly in a similar way. For example, for the case where $k_1<0$ and $|k_1|>m$ (*i.e.*, the CM mode is excited by blue-sideband laser beam applied to the first ion), the relevant evolution equations can be easily obtained from eq. (4) by making the replacements $|e_j\rangle \leftrightarrow |g_j\rangle$ in the third and fourth formulas.

Based on the conditional quantum dynamics derived above, we now show how to simultaneously manipulate quantum information stored in two ions by *a single* operation, *i.e.*, implementing the universal two-qubit gate and engineering two-qubit entanglement, beyond the LD limit. This is achieved by properly controlling the initial phases, wave vectors, and the duration of the applied simultaneous beams.

First, the two-qubit controlled gate implies that the effect of the operation on the second qubit (target one) depends on what state the first qubit (control one) is in. It is easily seen from eq. (4) that, if the following conditions:

$$\cos(\tilde{\alpha}_2\tau)=\sin(\lambda_+\tau)=\sin(\lambda_-\tau)=1, \tag{5}$$

are satisfied, the two-qubit controlled operation

$$\tilde{C}_{12}^X = |g_1\rangle|g_2\rangle\langle g_1|\langle g_2| + |g_1\rangle|e_2\rangle\langle g_1|\langle e_2| - ie^{-i\phi_2}|e_1\rangle|g_2\rangle\langle e_1|\langle e_2| - ie^{i\phi_2}|e_1\rangle|e_2\rangle\langle e_1|\langle g_2| \tag{6}$$

can be realized directly. Here τ is the duration of the two simultaneously applied pulses. The state of the information bus is unchanged during this operation. Physically, by coupling to the common information bus, *i.e.*, the CM vibrational quanta, two ions may entangle. In fact, eqs. (4) clearly show that the rotation of the atomic states of the second ion depend on the quantum state of the bus, although the bus quanta is not excited by the applied resonant laser beam on the second ion ($\omega_2=\omega_0$). Therefore, the evolving quantum state of the bus, due to evolution of the first ion driven by a non-resonant laser beam, correlates two separate ions. Once the relevant experimental parameters are set to satisfy the conditions (5), the two-qubit entangled gate (6) is implemented. This controlled operation is equivalent [18] to the exact CNOT gate: $\hat{C}_{12}^X = |g_1\rangle|g_2\rangle\langle g_1|\langle g_2|+|g_1\rangle|e_2\rangle\langle g_1|\langle e_2|+|e_1\rangle|g_2\rangle\langle e_1|\langle e_2|+|e_1\rangle|e_2\rangle\langle e_1|\langle g_2|$, except for a local rotation. One can easily check that both the small and large, as well as negative and positive values of the LD parameters may be chosen to satisfy the condition (5) for realizing the desired two-qubit controlled gate. Our approach does not assume the LD approximation. Thus, the present scheme operates outside the LD regime and η_j can be large.

Second, we show below that the entangled states of two trapped ions can also be produced deterministically outside the LD regime. In fact, it is seen from the dynamical evolution eq. (4) that the entanglement between two ions can also be easily engineered outside the LD limit. Beginning with the non-entangled initial state $|\psi_0\rangle=|m\rangle|g_1\rangle|g_2\rangle$, we now describe a convenient approach to do this engineering.

We now first apply a laser beam with frequency $\omega_1 = \omega_0$, initial phase ϕ_1, and duration t_1 to ion 1, and realize the following evolution:

$$|\psi_0\rangle \xrightarrow{\hat{R}_1(m,t_1)} \cos(\tilde{\alpha}_1 t_1)|m\rangle|g_1\rangle|g_2\rangle - ie^{-i\phi_1}\sin(\tilde{\alpha}_1 t_1)|m\rangle|e_1\rangle|g_2\rangle = |\psi_1\rangle, \tag{7}$$

with $\hat{R}_1(m,t_1)$ being a simple operation of rotating the spin state of the ion 1. The CM mode quanta is not excited and the spin state of ion 2 is unchanged during this process. Obviously, this evolution may also be implemented by using a pair of simultaneous laser beams with frequencies $\omega_1 = \omega_0$ and $\omega_2 = \omega_0 - k_2\nu$ ($k_2 > m$), respectively.

We then apply another pair of simultaneous laser beams with frequencies $\omega_2 = \omega_0$ and $\omega_1 = \omega_0 - k_1\nu$ ($k_1 > m$) to realize the two-qubit controlled operation $\tilde{C}_{12}^X$ introduced above. This lets the non-entangled state $|\psi_1\rangle$ evolve in the desired entangled state $|\psi_{12}(t)\rangle$:

$$|\psi_1\rangle \xrightarrow{\tilde{C}_{12}^X} U(t)|g_1\rangle|g_2\rangle + V(t)|e_1\rangle|e_2\rangle = |\psi_{12}(t)\rangle, \tag{8}$$

with $U(t_1) = \cos(\tilde{\alpha}_1 t_1)$, $V(t_1) = -e^{i(\phi_2-\phi_1)}\sin(\tilde{\alpha}_1 t_1)$. Interestingly, the generated entangled state reduces to the two-qubit maximally entangled states; *i.e.*, the corresponding EPR states: $|\Psi_{12}^{\pm}\rangle = (|g_1\rangle|g_2\rangle \pm |e_1\rangle|e_2\rangle)/\sqrt{2}$, if the experimental parameters, such as the duration t_1 and wave vector $\vec{\kappa}_1$ of the applied laser beam for realizing the single-qubit rotation $\hat{R}_1(m,t_1)$, are further set properly.

We now briefly discuss the experimental realizability of this proposal. Indeed, it is not difficult to properly set the relevant parameters for satisfying the condition (5). For example, the desired LD parameter [19] defined by

$$\eta_j = \sqrt{\hbar\kappa_j^2/(2MN\nu)}\cos\theta_j, \qquad \theta_j = \arccos\left(\vec{\kappa}_j \cdot \vec{z}_j/\kappa_j\right), \tag{9}$$

can be reached by conveniently controlling the wave vector $\vec{\kappa}_j$ of the applied laser beams. It might seem at first, from the condition (5), that the present scheme for realizing the desired gate operation cannot be easily implemented, as the relevant experimental parameters should be set accurately. For example, if the Rabi frequencies and LD parameters are set as $\Omega_1 = \Omega_2 = \Omega$, $\eta_1 = \pm 2.18403$, $\eta_2 = \pm 1.73205$, then the duration τ of the two applied simultaneous pulses should be set up accurately as $\Omega\tau = 56.3186$. A simple numerical analysis shows that the lowest probability of realizing the desired operation is still very high, even if the relevant parameters cannot be set exactly. For example, even if the duration of the applied laser pulses is set roughly such that $\Omega\tau \approx 56.3$ (56.0), which is 0.03% (0.57%) away from the exact solution of condition (5), one can realize the operation $\tilde{C}_{12}^X$ with a very high probability, *i.e.*, 99.998% (99.36%), via a single-step operation. Indeed, by testing other values we have proven that our predictions are very robust.

Finally, we note that the duration of the applied simultaneous pulses for realizing the above quantum controlled operation is not much longer than that for other schemes [4,6,8,12] operating in the LD regime. The duration for implementing the above operation is estimated as $\sim 10^{-4}$ seconds, of the same order of the gate speed operating in the LD regime [3], for $\Omega/2\pi \approx 225\,\mathrm{kHz}$ and $\nu = \omega_z \approx 7\,\mathrm{MHz}$ [5]. To excite only the chosen sidebands of the CM mode, the spectral width of the applied laser pulse has to be small. However, it is not so small as to affect the speed of the operations, since the separation between the CM mode and the other ones is sufficiently large, *e.g.*, $\nu_1 - \nu = (\sqrt{3}-1)\nu \sim \nu$. In particular, it is not difficult for the current laser-cooling technologies to cool the trapped ions to their motional ground state. In our proposal, the ions must be cold. However, in principle, the heating effect may

be suppressed by atom interferometry [13]. Therefore, the present scheme might be realizable in the near future.

In summary, we have proposed an efficient theoretical scheme for simultaneously manipulating two-qubit quantum information stored in the chosen two ions. Under certain conditions a universal two-qubit gate can be exactly realized by a *single-step* pulse process performed by simultaneously applying a pair of laser beams with different frequencies. By using this quantum operation, one may engineer the entanglement state between the two chosen ions. All operations proposed here operate *outside* the LD regime and do *not* involve quantum transitions to auxiliary atomic levels. It is expected that the present scheme may be extended to simultaneously manipulate three-, four- or multi-qubit quantum information and may be also extended to other systems besides trapped ions, *e.g.*, quantum dots on quantum linear supports [20], for quantum information processing.

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REFERENCES

[1] Cirac J. I. and Zoller P., *Phys. Rev. Lett.*, **74** (1995) 4091.
[2] Kielpinski D. *et al.*, *Science*, **291** (2001) 1013; Schmidt-Kaler F. *et al.*, *J. Mod. Opt.*, **47** (2000) 2573.
[3] Monroe C. *et al.*, *Phys. Rev. Lett.*, **75** (1995) 4714; Roos Ch. *et al.*, *Phys. Rev. Lett.*, **83** (1999) 4713.
[4] King B. E. *et al.*, *Phys. Rev. Lett.*, **81** (1998) 1525.
[5] Turchette Q. A. *et al.*, *Phys. Rev. Lett.*, **81** (1998) 3631; Morigi G. *et al.*, *Phys. Rev. A*, **59** (1999) 3797.
[6] Ngerl H. C. *et al.*, *Phys. Rev. A*, **60** (1999) 145; Sckett C. A. *et al.*, *Nature (London)*, **404** (2000) 256.
[7] Duan L. M., Cirac J. I. and Zoller P., *Science*, **292** (2001) 1695; Li L. X. and Guo G. C., *Phys. Rev. A*, **60** (1999) 696.
[8] Jonathan D. and Plenio M. B., *Phys. Rev. Lett.*, **87** (2001) 127901.
[9] Wei L. F., Liu S. Y. and Lei X. L., *Phys. Rev. A*, **65** (2002) 062316.
[10] Jonathan D. *et al.*, *Phys. Rev. A*, **62** (2000) 042307.
[11] Childs A. M. and Chuang I. L., *Phys. Rev. A*, **63** (2000) 012306.
[12] Mølmer K. and Sørensen A., *Phys. Rev. Lett.*, **82** (1999) 1835; Sørensen A. and Mølmer K., *Phys. Rev. Lett.*, **82** (1999) 1971; Svandal A. and Hansen J. P., *Phys. Rev. A*, **65** (2002) 033406; Sørensen A. and Mølmer K., *Phys. Rev. A*, **62** (1999) 022311.
[13] Poyatos J. F., Cirac J. I. and Zoller P., *Phys. Rev. Lett.*, **81** (1998) 1322.
[14] Wineland D. J. and Itano W. M., *Phys. Rev. A*, **20** (1979) 1521; Stevens D., Brochard J. and Steane A. M., *Phys. Rev. A*, **58** (1998) 2750; Steane A., *Appl. Phys. B*, **64** (1997) 623; James D. F. V., *Appl. Phys. B*, **66** (1998) 181.
[15] See, *e.g.*, the following reviews: Bennett C. H. and DiVincenzo D. P., *Nature*, **44** (2000) 247; Ekert A. and Josza R., *Rev. Mod. Phys.*, **68** (1996) 733; Steane A. M., *Rep. Prog. Phys.*, **61** (1998) 117.
[16] Solano E. *et al.*, *Phys. Rev. A*, **62** (2000) 021401; **59** (1999) R2539; **64** (2001) 024304.
[17] Solano E., de Matos Fiho R. L. and Zagury N., *Phys. Rev. Lett.*, **87** (2001) 060402.
[18] Monroe C. *et al.*, *Phys. Rev. A*, **55** (2001) R2489.
[19] Šašura M. and Bužek V., *J. Mod. Opt.*, **49** (2002) 1593.
[20] Brown K. R., Lidar D. A. and Whaley K. B., *Phys. Rev. A*, **65** (2001) 012307.